Mathematics Applications Year 12
Test 3 2019   Calculator Assumed

SHENTON COLLEGE

Name: .... Solutions ....

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Time Allowed: 50 minutes | Marks   /54 |

Materials allowed: Calculator, 1 A4 page of notes, formula sheet provided.
Attempt all questions.

−1 units   ,   −1 rounding

Question 1 [2, 2, 2, 2 marks]

(a) How much does $40 000 amount to if it is invested at 3.7% per annum, compounded

    (i) quarterly for one year?   (2 marks)

$$40000\left(1+\frac{0.037}{4}\right)^4$$

$$= \$41500.66$$

✓ correct use of compounding.
✓ correct for 1 year.

    (ii) monthly for 10 years?   (2 marks)

$$40000\left(1+\frac{0.037}{12}\right)^{120}$$

$$= \$57876.43$$

✓ correct compounding in months.
✓ correct for 10 years

(b) What is the effective rate of interest when $40 000 is invested at 3.7% per annum, compounded quarterly?   (2 marks)

$$E.I.R = 100\left(\left(1+\frac{0.037}{4}\right)^4 - 1\right)$$

$$= 3.752\%$$

✓ correct use of formula.
✓ correct rate.

(c) Compare the effective rate of interest found in (b) with the effective rate of interest when $40 000 is invested at 3.7% per annum, compounded every fortnight.   (2 marks)

$$3.752\% < 3.767\%$$

Part (c) E.I.R. is higher due to more compounding periods.

✓ calculates new rate.
✓ compares values.

Question 2 [2, 1, 1, 2, 2, marks]

The table and graph below show N, the number of calls per weekday to a new enquiry line, over a three week period, together with five-point moving averages, M.

Day	Week 1					Week 2					Week 3				
	M	T	W	T	F	M	T	W	T	F	M	T	W	T	F
T	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
N	48	38	30	38	41	51	39	34	39	44	55	42	39	44	46
M			39.0	39.6	39.8	40.6	(b)	41.4	42.2	42.8	43.8	44.8	45.2		
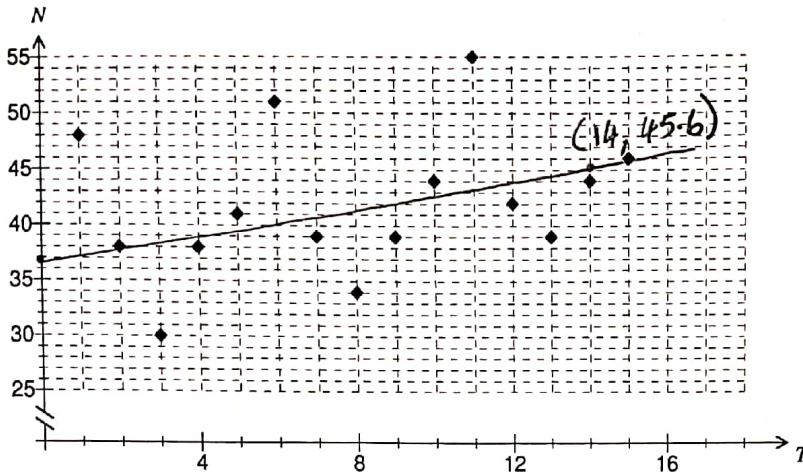
(a) i) Explain with detail why a five-point moving average is used. (2 marks)

There are five days (seasons) in 1 cycle, with a peak on Monday and a trough on Wednesday.

✓ explains period of the cycle.
✓ explains in context

ii) Explain why so many scores have no moving average value. (1 mark)

There is no data for previous week to make an average of 5 seasons.

✓ relevant explanation

(b) Write down the calculation that was used to determine the five-point moving average for Tuesday of Week 2. (1 mark)

$$M.A. = \frac{41 + 51 + 39 + 34 + 39}{5} = 40.8$$

✓ correct process and answer

(c) Determine the equation of the linear regression line that can be used to predict the moving average, M, from time, T. (2 marks)

$$M = 0.6327t + 36.76$$

✓ accurate gradient
✓ accurate equation

(d) Draw the line of regression calculated in (c) on the graph. (2 marks)

✓ correct intercept
✓ accuracy (using a point)

Question 3 [1, 2, 2, 1, 2 marks]

George, a young teacher, begins saving up for a deposit on a home. His savings account already has $5000 in it. Account interest helps to grow the balance of the account at a rate of 2.7% p.a, compounded monthly, and George deposits another $275 into the account at the end of every month.

The table below shows the amount in the account at the start of each month, the interest added, the next deposit, and the balance at the end of the month.

Month	Amount at the start of month	Interest for month	Deposit	Balance at the end of the month
1	5000.00	11.25	275.00	5286.25
2	5286.25	11.89	275.00	5573.14
3	5573.14	A	275.00	B
4				

(a) Give the monthly interest rate as a percentage. (1 mark)

0.225% ✓ correct rate

(b) Calculate the values of entries A and B in the table above. (2 marks)

$A = 12.54$ ✓ correct values

$B = 5860.68$ ✓ to 2 d.p.

(c) Write the recurrence relation used to determine the amount in the account at the end of each month. (2 marks)

$T_{n+1} = 1.00225 T_n + 275$

$T_0 = 5000$

OR $\quad T_{n+1} = \left(1 + \frac{0.027}{12}\right) T_n + 275$

✓ correct rule
✓ 1st term correct

(d) George has just made his ninth deposit.

(i) How much money is now in his account? (1 mark)

$\$7599.56$ ✓ correct balance

(ii) How much interest has he earned to date? (2 marks)

$7599.56 - \underset{(7475.56)}{\left(5000 + 9 \times 275\right)}$

$= \$124.56$

✓ finds correct amount George puts in his account
✓ subtracts correct values to determine interest.

Question 4 [2, 2 marks]

Casio has just released its seasonal index for share prices in June 2019 as being 0.941. John is considering purchasing shares of Casio for his shares portfolio. He knows that with shares, you "buy when the price is low and sell when it is high".

  a) Would you consider June to be a good month for John to buy shares in Casio? Justify your

  answer. (2 marks)

Yes, as its seasonal index is below the monthly average by approx. 6%

✓ Yes, with reason.

✓ accurate use of S.I.

  b) John purchased 100 shares for a total of $450. Determine the seasonally adjusted price for

  a single share of Casio in June 2019. (2 marks)

∴ $4.50/share

4.50 ÷ 0.941
 = $4.78 S.I.

✓ correctly adjusts

✓ correct price

Question 5 [1, 2, 2 marks]

Dave is wanting to buy a property in Albany for when he retires from teaching and is looking to buy a property in Merchant Street worth $540 000. He has a deposit of $30 000 and a local bank is willing to loan him the remainder at 4.3% p.a. compounded quarterly.
Dave feels he can make repayments of $9000 a quarter.

  (a) How much will he still owe after 6 years? (1 mark)

$414 272.34

✓ correct balance.

  (b) How many years will it take him to pay off the loan and what will be his final payment?

N 87.86
I 4.3
PV 510 000
PMT -9000
FV 0
P/Y 4
C/Y 4

88 quarters ∴ 22 years

and $7762.25 final payment.

(2 marks)

✓ correct years.

✓ final payment correct.

  (c) How much will Dave end up paying in total for the property when he has repaid the loan?

87 × 9000 + 7762.25 + 30 000

 = $820 762.25

(2 marks)

✓ correct number of repayments.

✓ correct total

Questions 6 [2, 3, 2, 2 marks]

Thunderbolt, a new electric car from the car manufacturer Tesla, can be purchased for $70000 brand new. It can depreciate at either
- 12% p.a. reducible balance **or**
- Flat rate depreciation is set at 9.6% p.a. **or**
- the expensive batteries that it runs on have a lifespan of 5000 hours before the car must be sold for a salvage cost of $22000.

(a) If the car depreciates at a flat rate, how much will the car be worth after 7 years?

(2 marks)

$$\$70000 \times 0.096 = \$6720$$

$$\therefore \ 70\,000 - 7 \times 6720 = \$22960$$

✓ determines flat rate value

✓ correct wourth after 7 years

(b) Tesla also claims that at some point in time, the reducible balance depreciated value will start giving higher resale values for the Thunderbolt than the flat rate depreciated value. After how many years will this happen and what is the difference in their value at this time?

(3 marks)

After 5 years

Reducible = $36941.23

Flat rate = $36400

$$\therefore \ Difference \ of \ \$541.23$$

✓ states time

✓ determines reducible value

✓ determines correct difference.

(c) Tesla does not like advertising it but realises that the battery life is a possible factor in the Thunderbolt's depreciation. If the car's value is based on battery life,

i) What is the loss in car value for every hour it is driven?

(2 marks)

$$70000 - 22000 = \$48000$$

$$\frac{48000}{5000} = \$9.60/hr$$

✓ determines loss due to battery loss

✓ correct loss per hour

ii) What would be the value of the car after one year if the customer drove 1400 hours.

(2 marks)

$$70000 - 1400 \times 9.60$$

$$= \$56560$$

✓ determines value loss for 1400 hours

✓ value of car after 1 year.

Question 7 [5, 2, 2, 3 marks]

A travelling group of famous comedians, The Monty Python Players, are performing four times a week at a local theatre. The audience number for the first 3 weeks of their shows is displayed in the table below.

Week	Day	Number in Audience (in hundreds)	Weekly average (to 2 d.p.)	Percentage of daily mean (to 2 d.p.)	Seasonally adjusted figures
One	Thurs	12		67.61	
	Fri	15	A	84.51	
	Sat	23		129.58	17.49
	Sun	21		E	
Two	Thurs	13		69.33	
	Fri	B	18.75	90.67	
	Sat	25		133.33	19.01
	Sun	20		106.67	
Three	Thurs	D		75.95	
	Fri	16	C	81.01	
	Sat	26		131.65	19.77
	Sun	22		111.39	

a) Determine the values of A, B, C, D and E in the table above.

(5 marks)

$A = 17.75$ $E = 118.31$

$B = 17$

$C = 19.75$

$D = 15$

✓ for each unknown.

b) The average seasonal index for Friday was found to be 85.40%. Find the average seasonal index for Saturday, clearly showing how you determined your answer.

(2 marks)

$$\frac{129.58 + 133.33 + 131.65}{3}$$

$$= 131.52\%$$

✓ shows process for determining index.
✓ correct index.

c) Seasonally adjust the audience numbers for Saturday only and record your answers in the last column in the table.

(2 marks)

✓ two correct
✓ all correct

d) The equation of the regression line, using t and the seasonally adjusted data, N, is

$N = 0.2587t + 17.15$

A financial consultant for the group wishes to determine the potential audience numbers for Friday of Week 4. Determine this prediction and comment on its reliability.

(3 marks)

$\hat{N} = 0.2587(14) + 17.15$

$= 20.7718$

20.7718×0.8540

$= 17.74$

≈ 1774 people

Reliable as extrapolated within 1 cycle (week)

✓ correct time value in equation
✓ correctly seasoned in hundreds
✓ reliable with reason.